

# Radiative viscosity of neutron stars

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## Abstract

We study non-linear effects of radiative viscosity of  $npe$  matter in neutron stars for both direct Urca process and modified Urca process, and find that non-linear effects will decrease the ratio of radiative viscosity to bulk viscosity from 1.5 to 0.5 (for direct Urca process) and 0.375 (for modified Urca process). Which means that for small oscillations of neutron star, the large fraction of oscillation energy is emitted as neutrinos; but for large enough ones, bulk viscous dissipation dominates.

*Key words:* neutron stars, bulk viscosity, radiative viscosity

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## 1. Introduction

Bulk viscosity can damp the density oscillations in compact stars, which could be excited at the time of their formation from supernovae explosions, or during the phase transitions [1], or due to instabilities result from the emission of gravitational waves [2, 3, 4, 5, 6]. As one of the most important transport coefficients, bulk viscosities of simple  $npe$  matter, of hyperon matter and even of quark matter, both in normal and superfluid states, have been extensively studied [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21], for more references see [22]. However, until recently, Sa'd et al. [23] demonstrated that there exists a new mechanism for dissipating the energy of stellar oscillations. They indicated that the mechanical energy of density perturbations is not only dissipated to heat via bulk viscosity, but also is radiated away via neutrinos. They named this new mechanism the radiative viscosity,

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and found it is 1.5 times larger than the bulk viscosity to all Urca processes, both in nuclear matter and quark matter.

The newly realized radiative viscosity was calculated only in the lowest order of  $\delta\mu/T$  [23], which corresponds to the linear approximation. However, the density oscillations during the stellar evolution may arise to sufficiently large amplitude so that no-linear effects can no longer be neglected. We aim to study the non-linear effects of radiative viscosity in this paper.

We will study non-linear effects of radiative viscosity of simple  $npe$  matter, both for direct Urca process ( $n \rightarrow p + e + \bar{\nu}_e$ ,  $p + e \rightarrow n + \nu_e$ ) and for modified Urca process ( $n + N \rightarrow p + N + e + \bar{\nu}_e$ ,  $p + N + e \rightarrow n + N + \nu_e$ ). As indicated by Lattimer et al. [24], if the proton and electron Fermi momenta are too small compared with neutron Fermi momenta, the nucleon direct Urca process is forbidden because it is impossible to satisfy conservation of momentum. Under typical conditions, one finds that the ratio of the number density of protons to that of nucleons must exceed 1/9 for the process to be allowed.

We not only make a numerical solution of radiative viscosity but also study the ratio of the viscosity coefficient to the bulk one. By doing so, we hope to know which mechanism is more important under special condition.

This paper is arranged as follows. In Sect.2, we derive the formulae of radiative viscosity including non-linear effects. And radiative viscosity for a specific model of neutron star (NS) matter is calculated in Sect.3. Finally, Sect.4 presents the conclusions and discussions.

## 2. The formulae of radiative viscosity

In this section we derive the expression for radiative viscosity of neutron stars whose cores consist of simple  $npe$  matter using an approach similar to that of bulk viscosity calculations has been done by Wang and Lu [11], Sawyer [12], Madsen [13] and Gupta et al. [14].

We first recall how to solve chemical potential perturbations with stellar oscillations. Assume that the volume per unit mass,  $v$ , changes periodically in time according to the relation

$$v(t) = v_0 + \Delta v \sin\left(\frac{2\pi t}{\tau}\right) = v_0 + \delta v(t), \quad (1)$$

where  $v_0$  is the equilibrium volume,  $\Delta v$  is the perturbation amplitude, and  $\tau$  is the period.

Taking the chemical potential difference as

$$\delta\mu \equiv \mu_p + \mu_e - \mu_n, \quad (2)$$

where  $\mu_n$ ,  $\mu_p$  and  $\mu_e$  are the chemical potentials of the neutrons, protons and electrons.  $\delta\mu$  can be expanded near the equilibrium according to

$$\delta\mu(t) = \left(\frac{\partial\delta\mu}{\partial v}\right)_0 \delta v + \left(\frac{\partial\delta\mu}{\partial n_p}\right)_0 \delta n_p + \left(\frac{\partial\delta\mu}{\partial n_n}\right)_0 \delta n_n + \left(\frac{\partial\delta\mu}{\partial n_e}\right)_0 \delta n_e, \quad (3)$$

(note that  $\delta\mu = 0$  in equilibrium)  $n_i$  are particle numbers per unit mass and

$$\delta n_p = -\delta n_n = \delta n_e = \int_0^t (dn_p/dt) dt. \quad (4)$$

Using the thermodynamical relations

$$\frac{\partial\mu_i}{\partial v} = -\frac{\partial P}{\partial n_i}, \quad (5)$$

and considering  $\rho_i = n_i/v_0$  ( $\rho_i$  is the particle numbers per unit volume), one gets

$$\delta\mu(t) = -A \frac{\Delta v}{v_0} \sin\left(\frac{2\pi t}{\tau}\right) + B \int_0^t (d\rho_p/dt) dt, \quad (6)$$

where

$$A = \left(\frac{\partial P}{\partial \rho_p}\right)_0 - \left(\frac{\partial P}{\partial \rho_n}\right)_0 + \left(\frac{\partial P}{\partial \rho_e}\right)_0, \quad (7)$$

$$B = \left(\frac{\partial\delta\mu}{\partial \rho_p}\right)_0 - \left(\frac{\partial\delta\mu}{\partial \rho_n}\right)_0 + \left(\frac{\partial\delta\mu}{\partial \rho_e}\right)_0, \quad (8)$$

and  $P(t)$  is the pressure, and the net reaction rate is

$$\frac{d\rho_p}{dt} = \Gamma_{\bar{\nu}} - \Gamma_{\nu} = -\Gamma(T, \delta\mu). \quad (9)$$

According to [25, 26] and introducing  $z = \delta\mu/\pi T$ , for direct Urca process

$$\Gamma_d(T, \delta\mu)\delta\mu = \epsilon_d(T, 0) \frac{714z^2 + 420z^4 + 42z^6}{457}, \quad (10)$$

$$\epsilon_d(T, 0) = 3.3 \times 10^{-14} \left( \frac{x_p \rho}{\rho_0} \right)^{1/3} T^6 \text{MeV}^5, \quad (11)$$

and for modified Urca process

$$\Gamma_m(T, \delta\mu) \delta\mu = \epsilon_m(T, 0) \frac{14,680z^2 + 7560z^4 + 840z^6 + 24z^8}{11,513}, \quad (12)$$

$$\epsilon_m(T, 0) = 3.6 \times 10^{-18} \left( \frac{x_p \rho}{\rho_0} \right)^{1/3} T^8 \text{MeV}^5, \quad (13)$$

where  $x_p = \rho_p/\rho_b$  is the ratio of the number density of protons to that of nucleons, and  $\rho_0 = 0.16 \text{fm}^{-3}$ .

Given a specific equation of state (EOS) of *npe* matter,  $\delta\mu(t)$  can be calculated from eqs. (6), (7), (8) and (9) numerically, which will be done in section 3. Once the time dependence of chemical potential difference is known, one can easily calculate the bulk and radiative viscous coefficient.

For a periodic process, the expansion and contraction of the system will induce not only the dissipation of oscillation energy to heat, but also the loss of oscillation energy through an increasing of the neutrino emissivity. Bulk viscous coefficient  $\zeta$  and radiative viscous coefficient  $\mathcal{R}$  can be defined for the description of these dissipation mechanisms, respectively [23]

$$\langle \dot{\mathcal{E}}_{\text{diss}} \rangle = -\frac{\zeta}{\tau} \int_0^\tau dt (\nabla \cdot \vec{v})^2, \quad (14)$$

$$\langle \dot{\mathcal{E}}_{\text{loss}} \rangle = \frac{\mathcal{R}}{\tau} \int_0^\tau dt (\nabla \cdot \vec{v})^2, \quad (15)$$

where  $\vec{v}$  is the hydrodynamic velocity associated with the density oscillations.

Using the continuity equation, one obtains

$$\zeta = -2\langle \dot{\mathcal{E}}_{\text{diss}} \rangle \left( \frac{v_0}{\Delta v} \right)^2 \left( \frac{\tau}{2\pi} \right)^2, \quad (16)$$

$$\mathcal{R} = 2\langle \dot{\mathcal{E}}_{\text{loss}} \rangle \left( \frac{v_0}{\Delta v} \right)^2 \left( \frac{\tau}{2\pi} \right)^2, \quad (17)$$

where the energy dissipation is

$$\langle \dot{\mathcal{E}}_{\text{diss}} \rangle = -\int_0^\tau \Gamma(T, \delta\mu) \delta\mu dt, \quad (18)$$

and the neutrino emissivity caused by the oscillation of  $\delta\mu$  is

$$\langle \dot{\mathcal{E}}_{\text{loss}} \rangle = \int_0^\tau [\epsilon(T, \delta\mu) - \epsilon(T, 0)] dt, \quad (19)$$

for direct Urca process [25]

$$\epsilon_d(T, \delta\mu) = \epsilon_d(T, 0) \left( 1 + \frac{1071z^2 + 315z^4 + 21z^6}{457} \right), \quad (20)$$

and for modified Urca process

$$\epsilon_m(T, \delta\mu) = \epsilon_m(T, 0) \left( 1 + \frac{22,020z^2 + 5640z^4 + 420z^6 + 9z^8}{11,513} \right). \quad (21)$$

### 3. Radiative viscosity for a specific model of NS matter

#### 3.1. EOS model

For illustration, we use a phenomenological EOS proposed by Prakash et al. [27]. According to these authors, the nuclear energy is presented in the form

$$E_N(\rho_b, x_p) = E_{N0}(\rho_b) + S(\rho_b)(1 - 2x_p)^2, \quad (22)$$

where  $E_N(\rho_b, x_p) = E_{N0}(\rho_b)$  ( $\rho_b, x_p = 1/2$ ) is the energy of symmetric nuclear matter and  $S(\rho_b)$  is the symmetry energy. Supposing the electron energy is  $E_e(\rho_b, x_p)$ , one has  $\partial[E_N(\rho_b, x_p) + E_e(\rho_b, x_p)]/\partial x_p = 0$  in  $\beta$  equilibrium.

At the saturation density  $\rho_0 = 0.16 \text{ fm}^{-3}$  the symmetry energy is measured in laboratory,  $S(\rho_0) = 30 \text{ MeV}$ ; while at higher  $\rho_b$  it is still unknown. Prakash et al. [27] displayed  $S(\rho_b)$  in the following form

$$S(\rho_b) = 13 \text{ MeV} [u^{2/3} - F(u)] + S(\rho_0)F(u), \quad (23)$$

where  $u = \rho_b/\rho_0$  and  $F(u)$  satisfies the condition  $F(1) = 1$ . We employ their model I ( $F(u) = u$ ) with the compression modulus of saturated nuclear matter  $K = 240 \text{ MeV}$ . This is a moderately stiff EOS, the maximum stellar mass for it is  $M = 1.977 M_\odot$ . The direct Urca process is allowed only if  $x_p > 1/9$ ; in our EOS model it demands  $\rho_b > 0.434 \text{ fm}^{-3}$ , which means the direct Urca process is completely forbidden in NS when its mass is smaller than  $M_D = 1.358 M_\odot$ .

Considering  $\rho_p = \rho_e = \rho_b x_p$  and  $\rho_n = \rho_b(1 - x_p)$ , one has

$$A = \frac{1}{\rho_b} \left( \frac{\partial P}{\partial x_p} \right)_0, \quad (24)$$

$$B = \frac{1}{\rho_b} \left( \frac{\partial \delta \mu}{\partial x_p} \right)_0. \quad (25)$$

Thus,  $\delta \mu(t)$  can be solved numerically, and the bulk and radiative viscosity can be calculated from eqs. (16) and (17), respectively.

### 3.2. The results

The solid line in Fig.1 and Fig.2 shows radiative viscosity as function of relative volume perturbation amplitude for  $\rho_b = 0.6 fm^{-3}$  and  $\rho_b = 0.3 fm^{-3}$ , note that the onset of the direct Urca process is  $\rho_b = 0.434$ . As indicated in ref. [14], the flat branches in Fig.1 for the direct Urca process follows the  $T^4$  law: when  $T$  increases by four orders (from  $10^{-4} MeV$  to  $1 MeV$ ), viscosity increases by about 16 orders. In contrast, the flat branches in Fig.2 for the modified Urca process follows the  $T^6$  law, and the radiative viscosity of modified Urca process is much lower than direct Urca process. However, the behavior of the onset of non-linear effect is similar for the two different processes: the lower the temperature, the smaller the relative volume perturbation amplitude is for non-linear effects to dominate. In the other words, as  $T$  decreases, non-linear effect becomes more and more important. Furthermore, we also present the results up to  $z^4$ . As we all know, when calculated up to  $z^2$ , one can only get the linear results; and non-linear effects are usually introduced by calculating up to  $z^4$ . But by comparing solid and dash curves in Fig.1 and Fig.2, we can see that it's not enough just including the lowest order of non-linear effects.

Fig.3 presents the ratio of radiative viscosity to bulk viscosity as function of relative volume perturbation amplitude. For both direct Urca and modified Urca,  $\mathcal{R}/\zeta = 1.5$  when  $\Delta v/v_0$  is sufficiently small, which is in agreement with the results given by Sa'd and Schaffner-Bielich [23]. And for both processes, non-linear effects lead to the decreasing of  $\mathcal{R}/\zeta$ , and the lower the temperature, the more remarkable the non-linear effect is. Nevertheless, for direct Urca process,  $\mathcal{R}/\zeta$  can reach 0.5 when  $\Delta v/v_0$  is large enough; but  $\mathcal{R}/\zeta$  can be equivalent to 0.375 for modified Urca process. Comparing the formulae (10) with (20), and (12) with (21), one immediately know that the

ratios only depend on the average value of the net reaction rate and the increments of neutrino emissivity due to off-equilibrium, and EOS can't effect the results at all. It is also interesting to plot  $\mathcal{R}/\zeta$  as function of  $\Delta\mu/T$  (Fig.4), where  $\Delta\mu$  is the perturbation amplitude of chemical potential difference  $\delta\mu(t)$ . It's undoubtable that the curves are overlapped for different values of temperature in Fig.4.

#### 4. Conclusions and discussions

We have studied the non-linear effects of radiative viscosity of simple *npe* matter, both for direct Urca process and for modified Urca process, and found that non-linear effects decrease  $\mathcal{R}/\zeta$  from 1.5 (the linear scenario) to 0.5 (for direct Urca process) and 0.375 (for modified Urca process). Which means that in the linear scenario, only  $\frac{2}{5}$  times of the total oscillation energy is converted to heat and the large faction of the energy is emitted as neutrinos; in contrast, if the amplitude of the oscillation is large enough and the non-linear effects cannot be ignored, the main part of the total oscillation energy is converted to heat (  $\frac{2}{3}$  for direct Urca process and  $\frac{8}{11}$  for modified process). Although the numerical results of the coefficients are calculated for a specific EOS, the ratio of the two types of viscosity coefficient is EOS-independent.

In the case of superfluid *npe* matter, since superfluidity causes different effects to the net reaction rate and the increments of neutrino emissivity due to off-equilibrium, we expect that both in linear regime and in non-linear regime, the value of  $\mathcal{R}/\zeta$  will have great differences comparing with the normal EOS. This is the further work we set about.

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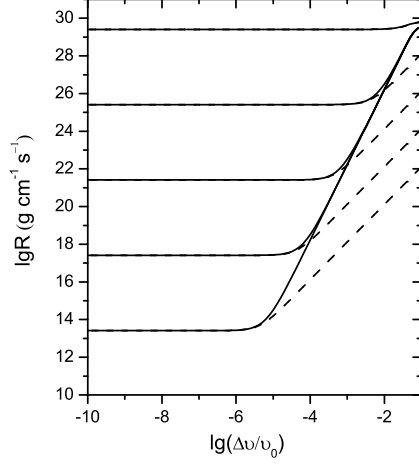


Figure 1: Radiative viscosity as function of relative volume perturbation amplitude for  $\tau = 10^{-3}\text{s}$  and  $\rho_b = 0.6 fm^{-3}$ , where the direct Urca process occurs. The dash curves are the results up to  $z^4$  and the solid curves up to  $z^6$ . The temperatures are  $10^{-4}$ ,  $10^{-3}$ ,  $10^{-2}$ ,  $10^{-1}$ , and 1 MeV from bottom to top, respectively.

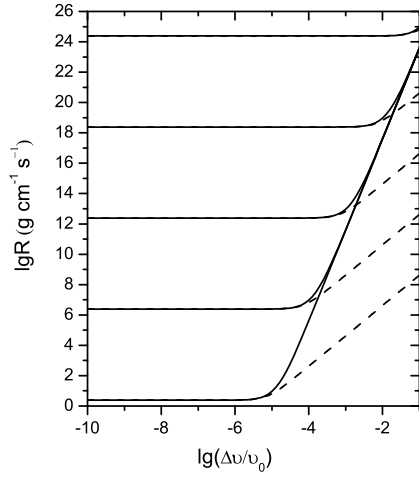


Figure 2: Radiative viscosity as function of relative volume perturbation amplitude for  $\tau = 10^{-3}\text{s}$  and  $\rho_b = 0.3 fm^{-3}$ , where the direct Urca process is forbidden. The dash curves are the results up to  $z^4$  and the solid curves up to  $z^8$ . The temperatures are  $10^{-4}$ ,  $10^{-3}$ ,  $10^{-2}$ ,  $10^{-1}$ , and 1 MeV from bottom to top.

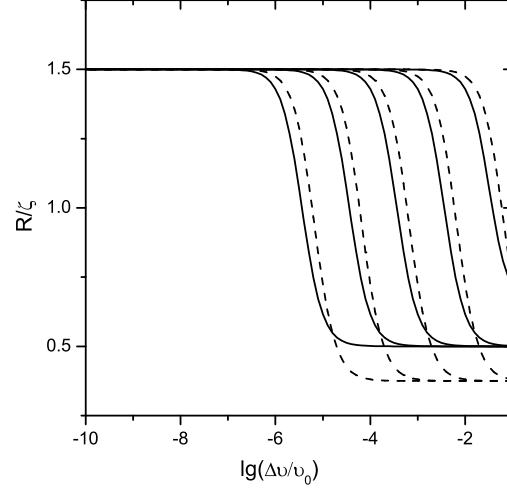


Figure 3:  $\mathcal{R}/\zeta$  as function of relative volume perturbation amplitude for  $\tau = 10^{-3}\text{s}$ ,  $\rho_b = 0.6\text{fm}^{-3}$  (solid curves) and  $\rho_b = 0.3\text{fm}^{-3}$  (dash curves). The temperatures are  $10^{-4}$ ,  $10^{-3}$ ,  $10^{-2}$ ,  $10^{-1}$ , and 1 MeV from left to right.

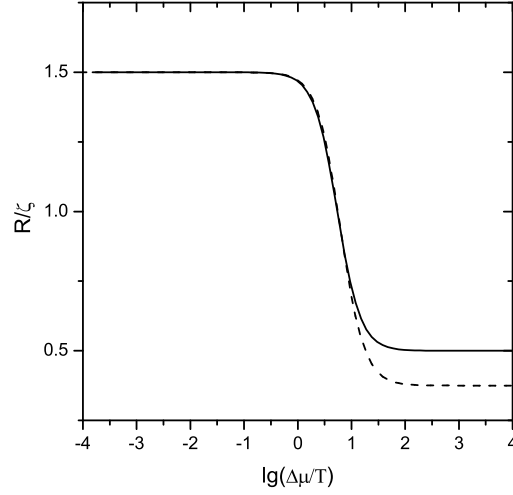


Figure 4:  $\mathcal{R}/\zeta$  as function of  $\lg(\Delta\mu/T)$  for  $\tau = 10^{-3}\text{s}$ ,  $\rho_b = 0.6\text{fm}^{-3}$  (solid curves) and  $\rho_b = 0.3\text{fm}^{-3}$  (dash curves), where  $\Delta\mu$  is the perturbation amplitude of chemical potential difference  $\delta\mu(t)$ .